PROBLEMS AND SOLUTIONS.

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PROBLEMS FOR SOLUTION.

[N.B. The editorial work of this department would be greatly facilitated if, on sending in problems, the proposers would also enclose their solutions—*when they have them.* If a problem proposed is not original the proposer is requested *invariably* to state the fact and to give an exact reference to the source.]

2863. Proposed by A. A. BENNETT, Baltimore, Md.

From their generation as roulette curves, show that the two hypocycloids of five cusps drawn with common vertices, are such that each is the envelope of a chord of constant length suitably placed upon the other.

Show that for any odd prime p, the $(p-1)/2$ distinct p-cusped hypocycloids with common vertices may be arranged in cycles, so that each is the envelope of a chord of constant length taken upon the succeeding curve of the cycle.

2864. Proposed by C. B. HALDEMAN, Ross, O.

If S is a side of a regular undecagon inscribed in a circle of radius unity, show that

$$S^5 - (S^4 - 3S^2 - 1)\sqrt{11} - 11S = 0.$$

2865. Proposed by JOSEPH ROSENBAUM, Milford, Conn.

In a circle, a chord AB is fixed in position and a moving chord CD is constant in length. Find the locus of the intersection of the bisectors of the angles ACD and BDC.

2866. Proposed by NORMAN ANNING, University of Michigan.

Equilateral triangles whose sides are 1, 3, 5, 7, $\cdots$ are placed so that their bases lie corner to corner in a straight line. Show that the vertices lie upon a parabola and are all at integral distances from its focus.

2867. Proposed by W. H. HAYS, Colton, Wash.

Show that the probable value of the sum of the squared differences, got by pairing two sets of n numbers each, at random, is $n(n^2-1)/6$; and that the coefficient of correlation in such a case is 0. See Thorndike, *Mental and Social Measurements*, page 164.

2868. Proposed by H. S. UHLER, Yale University.

Let the evolute of a given curve be called the evolute of the first order, let the evolute of the first evolute be called the evolute of the second order, etc. Then, being given the following parametric equations in which a is a constant and γ is the parameter, namely,

$$x = (1 + 2\sin^2\gamma)\cos\gamma - a\sin 2\gamma,$$

$$y = 2\sin^3\gamma + a\cos 2\gamma$$

find: (a) the parametric equations of the evolute of order n, both for n even and for n odd;
(b) a formula for the total length of the nth evolute;
(c) a formula for the total area of the nth evolute;
(d) the sum of the lengths of all the evolutes from; $n = 1$ to $n = \infty$; and
(e) the sum of the areas of all the evolutes from $n = 1$ to $n = \infty$.

Note: The original equations represent the envelope required in problem 2819, *1920*, 134.